Control Background

1. first-order system

$$\dot{y}(t) + \frac{1}{\tau} y(t) = u(t) \xRightarrow[\text{Transformation}]{\text{Laplace}} sY(s) + \frac{1}{\tau} Y(s) = U(s)$$

$$\boxed{\frac{Y(s)}{Y(s)} = \frac{\tau}{\tau s + 1}}$$

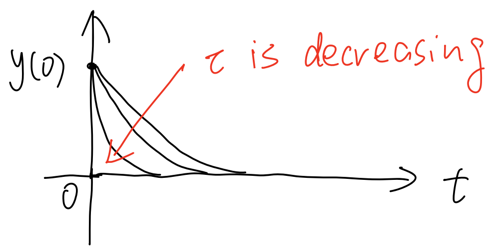
transfer Function

if $u(t) = 0$

the solution to the above first-ord system is

$$y(t) = e^{-(t/\tau)} \underbrace{y(0)}_{\text{initial condition}}$$

plot of the above solution for different τ's



2. second-order system

$$\ddot{y}(t) + 2\zeta\omega_n \dot{y}(t) + \omega_n^2 y(t) = u(t)$$

$$\Downarrow \text{ Laplace transformation}$$

$$s^2 Y(s) + 2\zeta\omega_n s Y(s) + \omega_n^2 Y(s) = U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

transfer function

two roots of characteristic equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

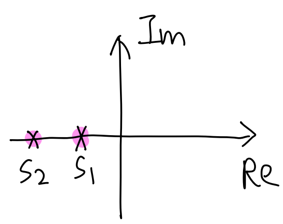
$$s_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2-1} \qquad s_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}$$

If $u(t) = 0$, the solution to the above second-order system is

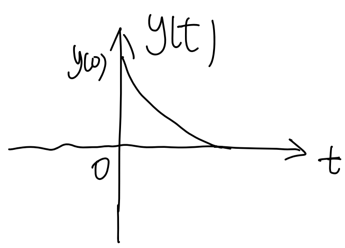
if $\zeta > 1$, this called overdamped

$$y(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t}$$

where c_1 and c_2 are determined by two initial conditions $y(0)$, $\dot{y}(0)$



root location

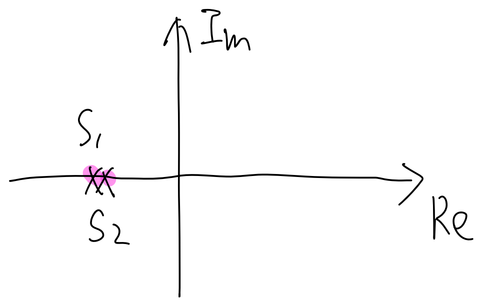


y(t) plot

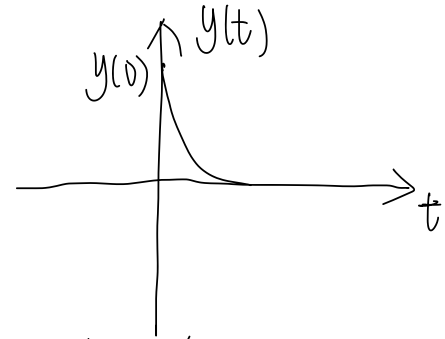
if $\zeta = 1$, this is called critically damped

$$y(t) = (c_1 + c_2 t)\, e^{-\omega_n t}$$

where c_1 and c_2 are determined by initial condition $y(0)$ and $\dot{y}(0)$



Root location

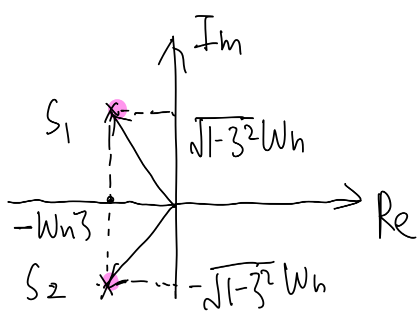


y(t) plot

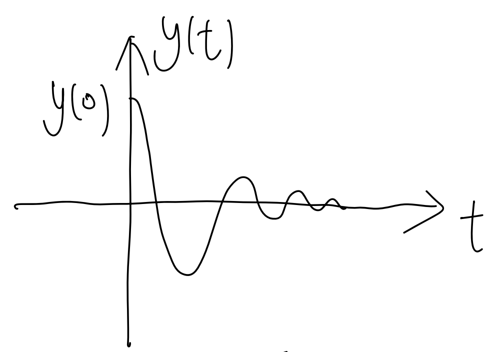
if $\zeta<1$, this called underdamped

$$y(t) = \left[d_1 \cos\left(\sqrt{1-\zeta^2}\,\omega_n t\right) + d_2 \sin\left(\sqrt{1-\zeta^2}\,\omega_n t\right)\right]e^{-\zeta\omega_n t}$$

where d_1 and d_2 will be determined by the initial conditions $y(0)$ and $\dot{y}(0)$

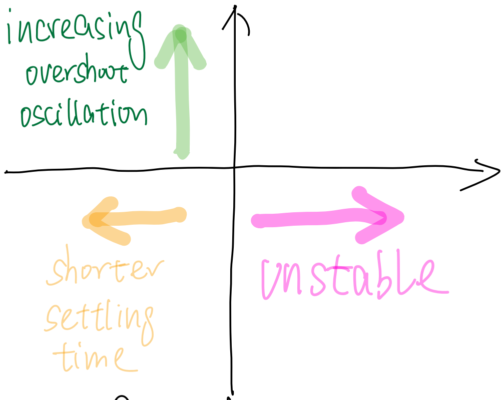


Root Location

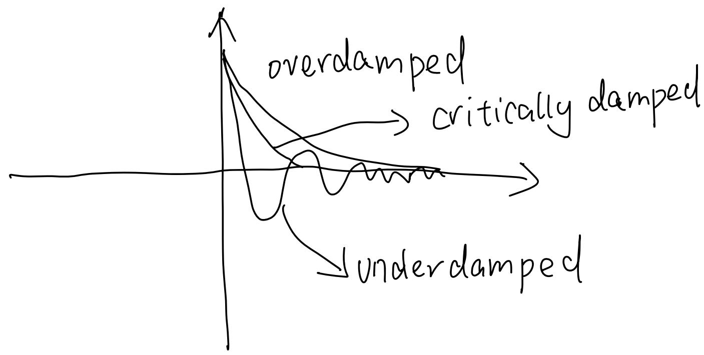


y(t) plot

Summary of the above three cases

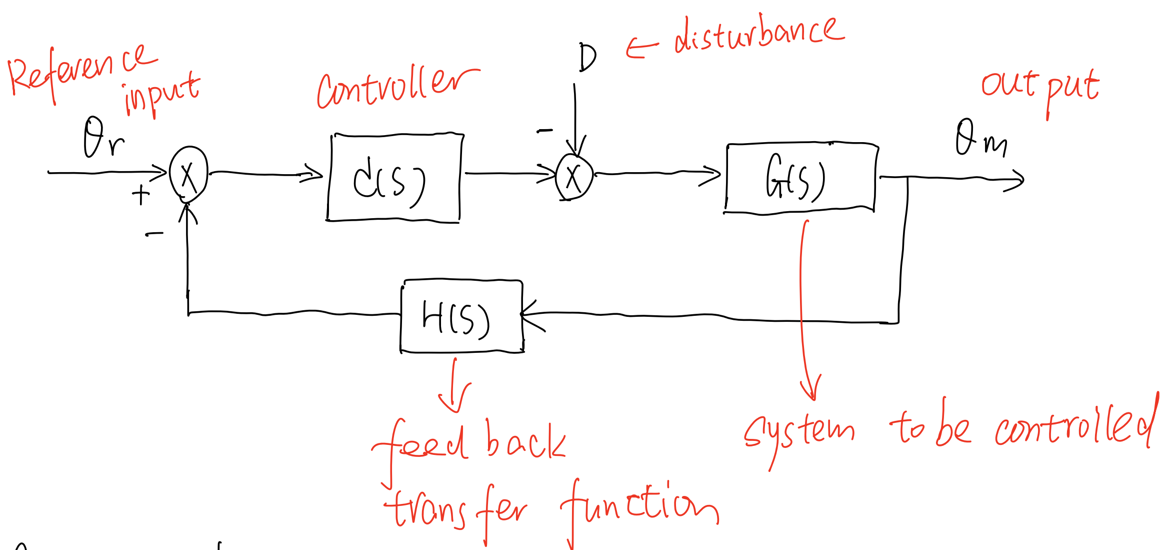


Root locations



y(t) plots

3. General Control Diagram (in s-domain)



Controller choices

- Proportional (P) controller

$$d(s) = K_p$$

- proportional - integral (PI) Controller

$$C(s) = K_p + K_i \frac{1}{s}$$

- proportional - integral - differential (PID) Controller

$$C(s) = K_p + K_i \frac{1}{s} + K_D s$$

Forward pass transfer Function (TF):

$$P(s) = C(s)\, G(s)$$

Backword pass transfer Function (TF):

$$H(s)$$

Open-loop TF:

$$P(s)\, H(s)$$

closed-Loop TF from θ_r to θ_m

$$\frac{\theta_m(s)}{\theta_r(s)} = \frac{P(s)}{1 + P(s)\, H(s)}$$

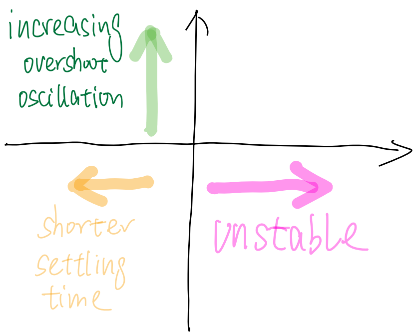
closed-Loop TF from D to θ_m

$$\frac{\theta_m(s)}{D(s)} = \frac{G(s)}{1 + P(s)\, H(s)}$$

characteristic Equation

$$1 + P(S)\,H(S) = 0$$

The relationship between the Roots of characteristic equation and the performace of the closed-Loop system:



Root Locus method is trying to find the relationship between open-loop gain K (increasing from $0 \to \infty$) and change of Location of Roots of characteristic Equation

open Loop gain K is from

$$P(S)\,H(S) = \frac{\boxed{K}\,(S-Z_1)(S-Z_2)\cdots(S-Z_m)}{(S-P_1)(S-P_2)\cdots(S-P_n)}$$

Location of roots of characteristic equation

$$1 + P(S)\,H(S) = (S-\boxed{S_1})(S-\boxed{S_2})\cdots(S-\boxed{S_n}) = 0$$

please see the youtobe Link sent for how to do.